

Final Focus System for a Muon Collider: A Test Model

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Abstract

The present scenario for a high luminosity 4 TeV on center of mass muon collider requires a beta function $\beta^* \approx 3$ mm at the interaction point. We discuss a *test model* of a basic layout which satisfies the requirements although it is not fully realistic.

INTRODUCTION

The parameters for a high luminosity, high energy muon collider are summarize in Tb.I. The design of an extreme low-beta interaction region for a muon collider [1] is non trivial and present a challenge in many ways similar to the one encounter in the Next Linear Collider (NLC) [2]. J. Erwin [3] and collaborators have designed the final focus system(FFS) for the NLC with $\beta_x^* \approx 37$ mm, $\beta_y^* \approx 100 \mu\text{m}$ and transverse beam dimensions of $\sigma_x \approx 420$ nm and $\sigma_y \approx 2.5$ nm for the 1 TeV center of mass case.

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Similarly, the latest version of the CLIC [4] FFS also calls for $\sigma_x \approx 90$ nm, $\sigma_y \approx 8$ nm and beta functions $\beta_x^* \approx 2.2$ mm, $\beta_y^* \approx 0.157$ mm at the interaction point (IP) for 500 GeV in the center of mass.

Both these designs of a FFS follow the prescription proposed by Brown [5]; it consists of two telescopes with two chromatic correction sections between them. The extremely low beta function at the IP results in the need of very strong quadrupoles which generate large chromaticity. This chromaticity must be corrected locally and this is achieved with two strong pairs of non-interleaved sextupoles. One pair, situated at the position of maximum β_x corrects the horizontal chromaticity, the other pair at maximum β_y corrects the vertical chromaticity. The two sextupoles of a pair are separated by a phase advance $\phi = \pi$ ($\Delta Q = -0.5$). This arrangement cancels the second-order geometric aberrations of the sextupoles thus reducing the second order tune shift by several order of magnitude. The bandwidth of the system is limited by the third-order aberrations and the remaining second-order amplitude dependent tune shift, [6]

$$\begin{aligned}\Delta Q_x &= \frac{\partial Q_x}{\partial \epsilon_x} \epsilon_x + \frac{\partial Q_x}{\partial \epsilon_y} \epsilon_y \\ \Delta Q_y &= \frac{\partial Q_y}{\partial \epsilon_x} \epsilon_x + \frac{\partial Q_y}{\partial \epsilon_y} \epsilon_y\end{aligned}\tag{1}$$

These aberrations arise from: a) small phase error between the sextupoles and the final quadruplet; b) finite length of the sextupoles.

The residual chromaticity at the IP could be reduced by adding a number of sextupoles at locations with nonzero dispersion, as it was suggested by Brinkmann [7], which have the function of correcting locally the chromaticity of each module. Finally, a system of octupoles could be designed to correct the third-order aberrations. Overall, it is believed that it could be possible to construct a system with a bandwidth of $\approx 1\%$.

There have been several previous attempts to design the FFS for a muon collider [8], which have been summarized and compared in ref. [9].

I. RESULTS

Following the above prescription, a design by Napoly [10] was taken as a starting point; the final doublet used by Napoly was replaced by a quadruplet as the final telescope [11]. Partial optimization of the design has been performed with the code MAD, alpha VMS version 81.6/1 [12]. Another important modification was to replace the split sextupoles by single elements, this simple change reduced the amplitude dependent tune shift by an order of magnitude.

Starting from the interaction point (IP), there is an initial telescope with magnification 3, ending in a focus O_1 (see Fig.1). Then it follows by FODO cells, each with a phase

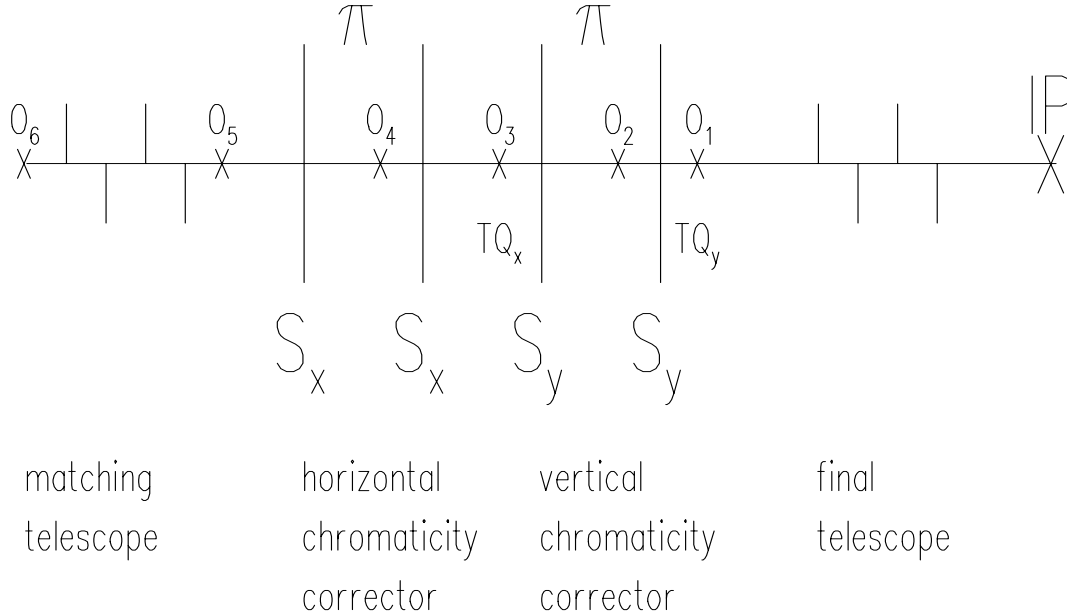


FIG. 1. Schematic of a FFS with extremely small beta function at the IP for a muon collider advance of exactly $\frac{\pi}{2}$. Intermediate foci are generated at O_2 , O_3 . Approximately midway between these foci, vertical correction sextupoles (S_{y1} and S_{y2}) are introduced and are near maxima in β_y . Then follow another similar sequence of cells with intermediate foci at O_4 , O_5 , but in these cells the sign of all quadrupoles have been reversed. The two following sextupoles, placed between these foci now fall on maxima of β_x and thus serve to correct the horizontal chromaticity ($\frac{\partial Q}{\partial \delta}$). The horizontal bending magnets are introduced to achieve dispersion at the sextupoles; reverse bends are also used to reduce the dispersion between

the vertical correction sextupoles and thus avoid otherwise excessive second order dispersion ($x \approx \delta^2$). The strength of the sextupoles (S_x and S_y) are adjusted to minimize the first order chromaticity while trim quadrupoles (TQ_x and TQ_y) are used to minimize the second order chromaticity ($\frac{\partial^2 Q}{\partial \delta^2}$). The lattice is ended by a second telescope, also with magnification 3, that could be used to match the correction system into an arc lattice. The final focus system, from the matching telescope to the IP [11], has a very small residual chromaticity and should be chromatically transparent when is attached to the arc lattice.

The total length of the FFS is ≈ 475.8 m. The lattice consists of 44 quadrupoles, 14 sector dipoles and 4 sextupoles. The lattice does, however, include dipoles with excessive field and with no space between most elements.

The variation of the tune shift at the IP as a function of $\delta = \frac{\delta p}{p}$ is shown in Fig.2. Q_y is essentially flat over a bandwidth of $\pm 0.4\%$; Q_x has obvious non-linear components, although the variation of tune, peak to peak is less than 0.03 within a bandwidth of $\pm 0.3\%$. Likewise, the relative β^* variation ($\Delta\beta^* = \frac{(\beta^*-3)}{3}$) (Fig.3) is negligible within a bandwidth of $\pm 0.3\%$. The remaining figures show the beta functions as a function of the position z (Fig.4a); the chromaticity (Fig.4b) and dispersion (Fig.4c) as function of position z along the FFS and energy spread δ (Fig.4d).

II. SUMMARY

A *test model* of the FFS for a muon collider has been described. The design satisfies the collider requirements, although it is not fully realistic. Errors and tolerance analysis are yet to be performed as well as tracking through the FFS to conform the achievable luminosity.

In order to make this final focus realistic, spaces will have to be introduced between elements and its length will have to be increased to achieve the require dispersion without unrealistic dipole fields.

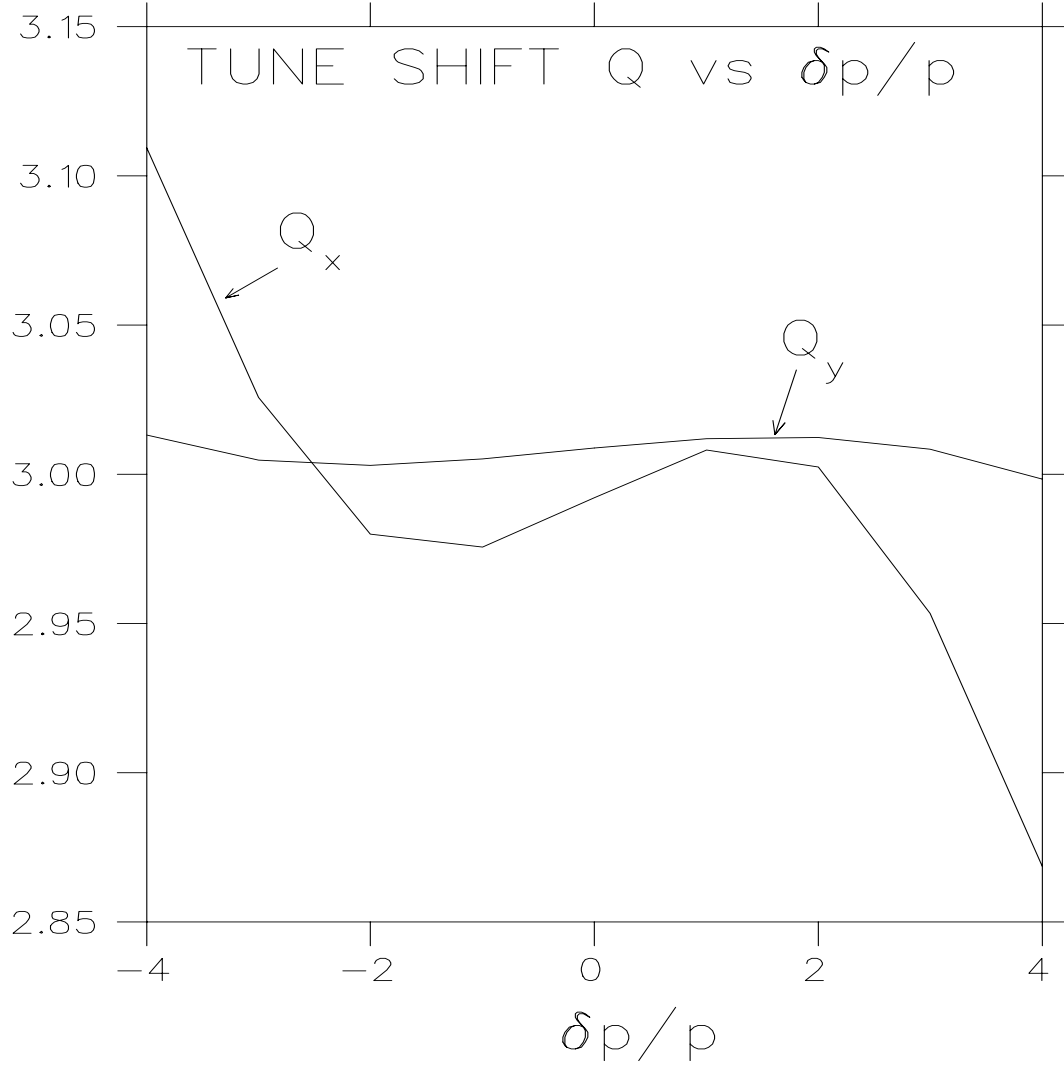


FIG. 2. Tune shift $Q_{x,y}$ vs $\frac{\delta p}{p}$

We would like to emphasize the need of new levels of sophistication in the correction of non-linear tune shift both in amplitude and momentum dependency, in storage rings with extreme low beta functions at the IP.

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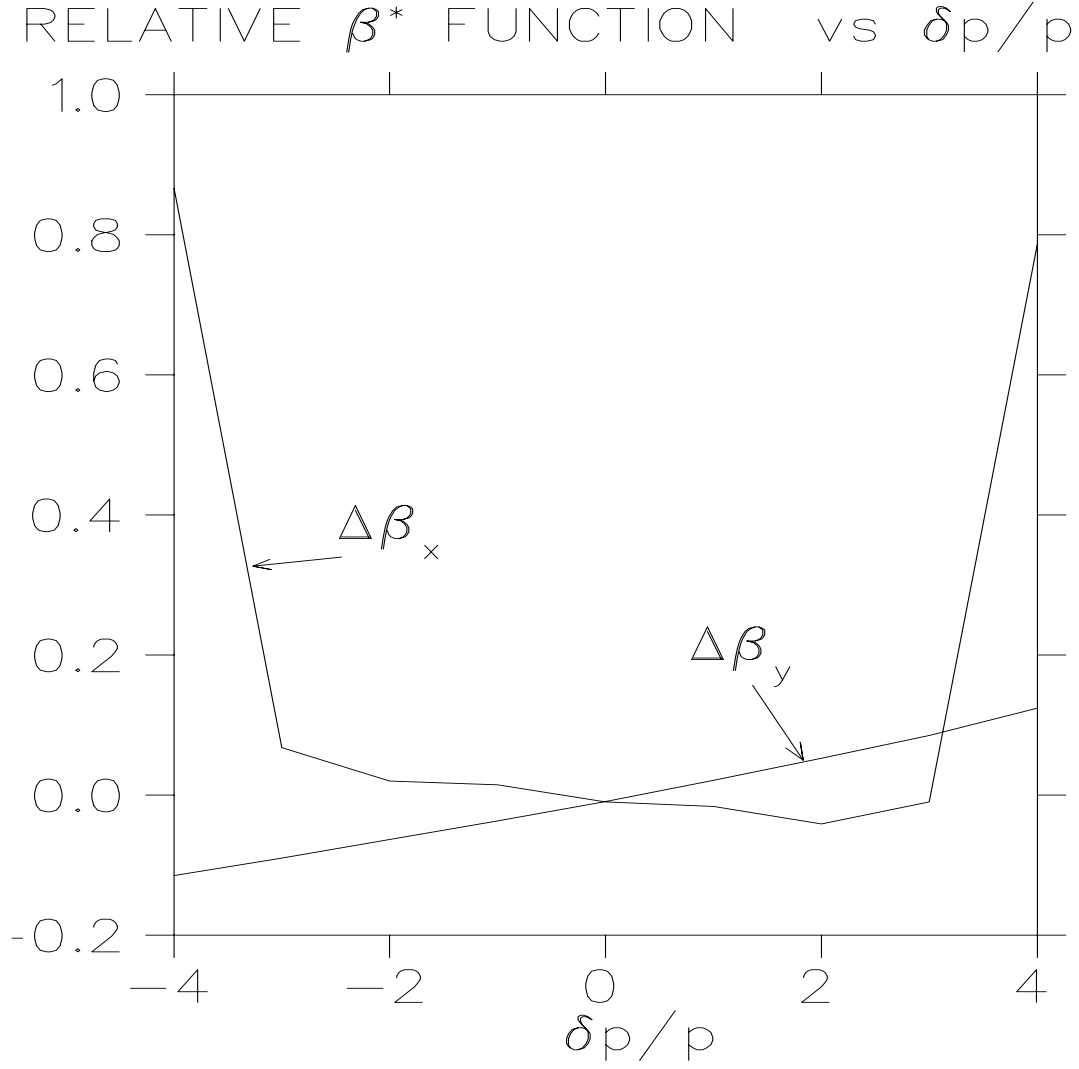


FIG. 3. Beta function β^* vs $\frac{\delta p}{p}$

and O. Napoly; both authors thanks K.-Y. Ng and D. Trbojevic for their helpful comments and W. Graves for his contribution in the early stage of this work.

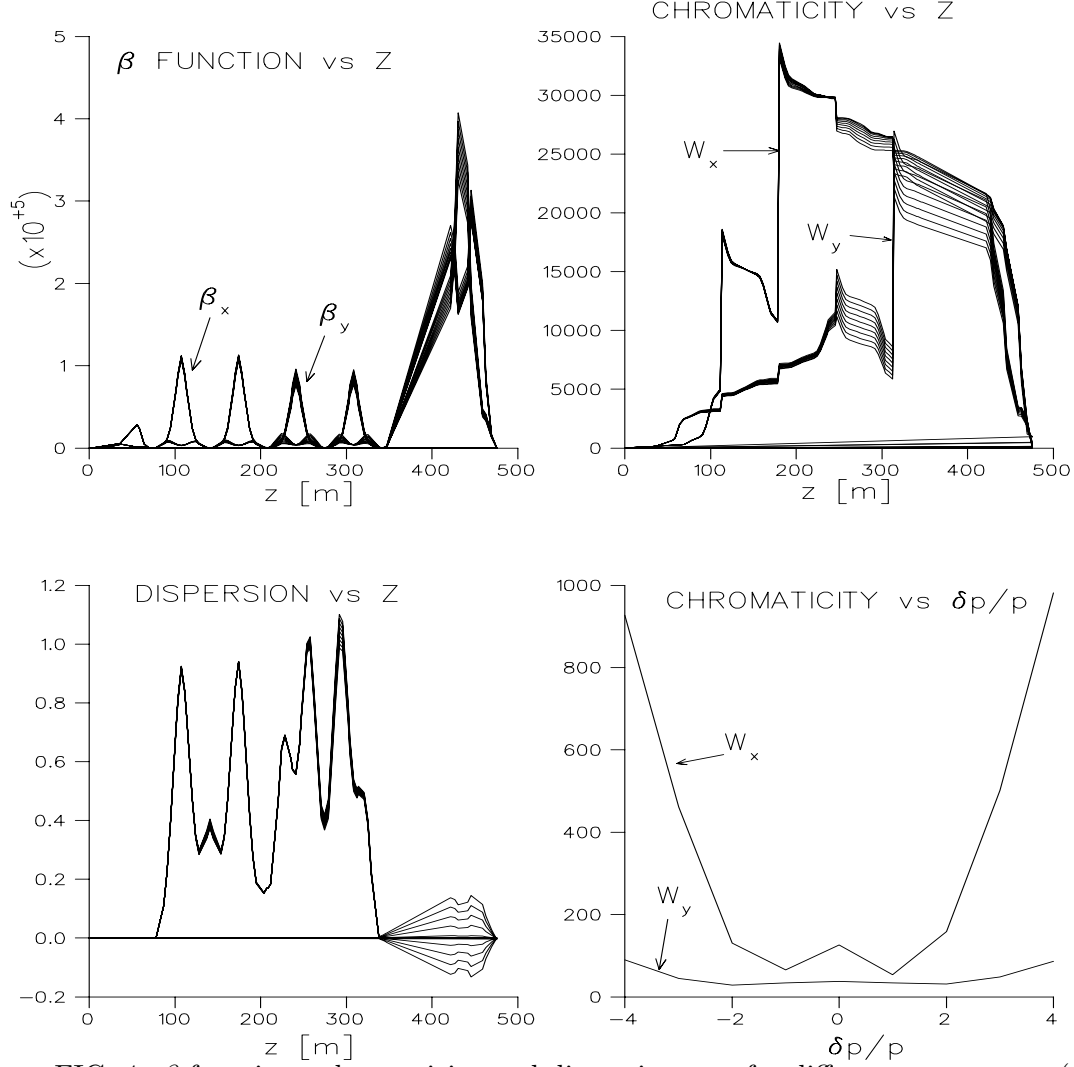


FIG. 4. β functions, chromaticity and dispersion vs z for different momentum ($\pm 0.4\%$); lower right window: chromaticity vs $\frac{\delta p}{p}$

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TABLE I. HIGH ENERGY-HIGH LUMINOSITY $\mu^+ \mu^-$ COLLIDER

	2 TeV	250 GeV
Maximum c-of-m Energy [TeV]	4	0.5
Luminosity \mathcal{L} [$10^{35}\text{cm}^{-2}\text{s}^{-1}$]	1.0	0.05
Time Between Collisions [μs]	12	1.5
Energy Spread σ_e [units 10^{-3}]	2	2
Pulse length σ_z [mm]	3	8
Beam Radius at the \mathcal{IP} [10^{-6}m]	2.8	16.0
Free space at the \mathcal{IP} [m]	6.25	2.0
Luminosity life time [s]/No.turns	0.02/900	0.003/800
<i>rms</i> Norm. emittance, $\epsilon_{x,y}^N$ [10^{-6}m-rad]	50.0	80.0
<i>rms</i> Unnorm. emittance, $\epsilon_{x,y}$ [10^{-6}m-rad]	0.0026	0.034
Beta Function at \mathcal{IP} , β^* [mm]	3	8
<i>rms</i> Beam size at \mathcal{IP} [μm]	2.8	16.
Focus Pole field at IP [T]	6.0	6.0
Maximum Beta Function, β_{max} [km]	400	14
Momentum Compaction, α [units 10^{-4}]	1.5	2.0
Magnet Aperture closest \mathcal{IP} [cm]	12	11.25
Beam-Beam tune shift per crossing	0.04	0.04
Repetition Rate [Hz]	15	15
RF frequency [GHz]	3	1.3
RF voltage [MeV]	1500	140
Particles per Bunch [units 10^{12}]	2	4
No. of Bunches of each sign	2	1
Peak current $\mathcal{I} = \frac{eNc}{\sqrt{2\pi}\sigma_z}$ [kA]	12.8	9.6
Average current $\mathcal{I} = \frac{eNc}{\text{Circum}}$ [A]	0.032	0.25
Circumference [km]	7	0.9
Peak Magnetic Field [T]	9	9
Average Magnetic Field [T]	6	6